INDEFINITE INTEGRATION

ELEMENTARY EXERCISE

Sol.1 $\int 2^x.e^x dx$

$\Rightarrow \int (2e)^x dx \Rightarrow \frac{(2e)^x}{\ln 2e} + C \Rightarrow \frac{2^x.e^x}{1+\ln e} + C$

Sol.2 $\int \frac{1+\cos^2 x}{1+\cos 2x} dx$

$\Rightarrow \int \frac{1+\cos^2 x}{2\cos^2 x} \Rightarrow \frac{1}{2} \int (\sec^2 x + 1) dx$

$\Rightarrow \frac{1}{2} (\tan x + x) + C$

Sol.3 $\int \frac{1-\tan^2 x}{1+\tan^2 x} dx$

$\Rightarrow \int \frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x} dx \Rightarrow \int \frac{\cos 2x}{1} \Rightarrow \frac{1}{2} \sin 2x + C$

Sol.4 $\int \frac{1+\tan^2 x}{1+\cot^2 x} dx$

$\Rightarrow \int \frac{\sec^2 x}{\text{cosec}^2 x} dx \Rightarrow \int \frac{\sin^2 x}{\cos^2 x} dx \Rightarrow \int \frac{1-\cos^2 x}{\cos^2 x} dx$

$\Rightarrow \tan x - x + C$

Sol.5 $\int \frac{e^{5\ell nx} - e^{4\ell nx}}{e^{3\ell nx} - e^{2\ell nx}} dx$

$\Rightarrow \int \frac{x^5 - x^4}{x^3 - x^2} dx \Rightarrow \int \frac{x^4(x-1)}{x^2(x-1)} dx \Rightarrow \int x^2 dx$

$\Rightarrow \frac{x^3}{3} + c$

Sol.6 $\int (e^{a\ell nx} + e^{x\ell na}) dx (a > 0)$

$\Rightarrow \int (x^a + a^x) dx \Rightarrow \frac{x^{a+1}}{a+1} + \frac{a^x}{\ln a} + C$

Sol.7 $\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx$

$\Rightarrow \int \frac{\cos^2 x - \sin^2 x}{\cos^2 x . \sin^2 x} dx \Rightarrow \int (\text{cosec}^2 x - \sec^2 x) dx$

$\Rightarrow -(\cot x + \tan x) + C$

Sol.8 $\int \frac{1+2x^2}{x^2(1+x^2)} dx$

$\Rightarrow \int \left[\frac{1+x^2}{x^2(1+x^2)} + \frac{x^2}{x^2(1+x^2)} \right] dx$

$\Rightarrow \int \frac{1}{x^2} dx + \int \frac{1}{1+x^2} dx \Rightarrow -\frac{1}{x} + \tan^{-1} x + C$

Sol.9 $\int 4\cos\frac{x}{2}.\cos x.\sin\frac{21}{2} x dx$

$\Rightarrow \int 2\cos x(\sin 11x + \sin 10x) dx$

$\Rightarrow \int 2\cos x.\sin 11x dx + \int 2\cos x.\sin 10x dx$

$\Rightarrow \int [\sin 12x + \sin 10x] dx + \int [\sin 11x + \sin 9x] dx$

$\Rightarrow -\left[\frac{\sin 12x}{12} + \frac{\sin 10x}{10} + \frac{\sin 11x}{11} + \frac{\sin 9x}{9} \right] + C$

Sol.10 $\int \frac{\cos x - \sin x}{\cos x + \sin x} (2 + 2\sin 2x) dx$

$\Rightarrow \int \frac{2.(\cos x - \sin x)}{(\cos x + \sin x)} .(\cos x + \sin x)^2 dx$

$\Rightarrow \int 2(\cos^2 x - \sin^2 x) dx \Rightarrow 2\int \cos 2x \, dx$

$\Rightarrow \sin 2x + C$

Sol.11 $\int (3\sin x \cos^2 x - \sin^3 x) dx$

$\Rightarrow \int \sin x(3\cos^2 x - \sin^2 x) dx$

$\Rightarrow \int \sin x(4\cos^2 x - 1) dx \Rightarrow$ put $\cos x = t$

$\therefore -\sin x \, dx = dt$ or $\sin x dx = (-dt)$

$\Rightarrow \int (1 - 4t^2) dt \Rightarrow t - \frac{4}{3} t^3 + c$

put $t = \cos x \Rightarrow \cos x - \frac{4}{3} (\cos x)^3 + c$

Sol.12 $\int \cos x^\circ dx$

$\Rightarrow \int \cos\left(\frac{\pi}{180}x\right)dx \Rightarrow \frac{-180}{\pi}\sin\left(\frac{\pi}{180}x\right) + C$

$\Rightarrow \frac{-180}{\pi}\sin x^\circ + c$

Sol.13 $\int \frac{(1+x)^2}{x(1+x^2)}dx$

$\Rightarrow \int \frac{1+x^2+2x}{x(1+x^2)}dx \Rightarrow \int\left[\frac{1+x^2}{x(1+x^2)} + \frac{2x}{x(1+x^2)}\right]dx$

$\Rightarrow \int \frac{1}{x}dx + 2\int \frac{1}{1+x2}dx \Rightarrow \ln|x| + 2\tan^{-1}x + c$

Sol.14 $\int \frac{x}{2x+1}dx$

$\Rightarrow \frac{1}{2}\int \frac{2x+1-1}{2x+1}dx \Rightarrow \frac{1}{2}\int\left[\frac{2x+1}{2x+1} - \frac{1}{2x+1}\right]dx$

$\Rightarrow \frac{1}{2}\left[x - \frac{1}{2}\ln|2x+1|\right] + c$

Sol.15 $\int \frac{\sec 2x - 1}{\sec 2x + 1}dx$

$\Rightarrow \int \frac{1-\cos 2x}{1+\cos 2x}dx \Rightarrow \int \frac{1-\cos^2 x}{\cos^2 x}dx$

$\Rightarrow \int (\sec^2 x - 1)dx \Rightarrow \tan x - x + C$

Sol.16 $\int \frac{\sin x + \cos x}{\sqrt{1+\sin 2x}}dx (\cos x + \sin x > 0)$

$\Rightarrow \int \frac{\sin x + \cos x}{(\cos x + \sin x)^2}dx \Rightarrow \int \frac{\sin x + \cos x}{|\cos x + \sin x|}dx$

$\Rightarrow \int \frac{\sin x + \cos x}{\cos x + \sin x}dx \quad [\because \cos x + \sin x > 0]$

$\Rightarrow x + c$

Sol.17 $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha}dx$

$\Rightarrow \int \frac{(2\cos^2 x - 1) - (2\cos^2 \alpha - 1)}{\cos x - \cos \alpha}dx$

$\Rightarrow \int \frac{2(\cos^2 x - \cos^2 \alpha)}{(\cos x - \cos \alpha)}dx \Rightarrow 2\int (\cos x + \cos \alpha)dx$

$\Rightarrow 2(\sin x + x.\cos \alpha) + c$

Sol.18 $\int \frac{x^6 - 1}{x^2 + 1}dx$

upon dividing, we get

$\Rightarrow \int\left[\left(x^4 - x^2 + 1\right) - \frac{2}{(1+x^2)}\right]dx$

$\Rightarrow \frac{x^5}{5} - \frac{x^3}{3} + x - 2\tan^{-1}x + c$

Sol.19 $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}dx$

$\Rightarrow \int (\tan x.\sec x + \cot x \operatorname{cosec} x)dx$

$\Rightarrow \sec x - \operatorname{cosec} x + C$

Sol.20 $\int \frac{x^4 + x^2 + 1}{2(1+x^2)}dx$

$\Rightarrow \int \frac{x^2(x^2+1)+1}{2(1+x^2)} = \int\left[\frac{x^2}{2} + \frac{1}{2(1+x^2)}\right]dx$

$\Rightarrow \frac{x^3}{6} + \frac{1}{2}\tan^{-1}x + c$

Sol.21 $\int \sqrt{1-\sin 2x}\,dx$

$\Rightarrow \int \sqrt{\sin^2 x + \cos^2 x - 2\sin x.\cos x}\,dx$

$\Rightarrow \int \sqrt{(\sin x - \cos x)^2}\,dx$

$\Rightarrow (\sin x + \cos x)\,\text{sgn}\,(\cos x - \sin x) + c$

Sol.22 $\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x.\cos^2 x}dx$

$\Rightarrow \int \frac{(\sin^2 x + \cos^2 x)(\sin^4 x + \cos^4 x - \sin^2 x\cos^2 x)}{\sin^2 x.\cos^2 x}dx$

$\Rightarrow \int \frac{(\sin^2 x + \cos^2 x)^2 - 3\sin^2 x\cos^2 x}{\sin^2 x.\cos^2 x}$

$\Rightarrow \int \frac{1 - 3\sin^2 x\cos^2 x}{\sin^2 x.\cos^2 x}dx$

$\Rightarrow \int \left[\sec^2 x + \operatorname{cosec}^2 x - 3\right]dx$

$\Rightarrow \tan x - \cot x - 3x + C$

Sol.23 $\int \frac{(\sqrt{x}+1)(x^2-\sqrt{x})}{x\sqrt{x}+x+\sqrt{x}}\,dx$

put $\sqrt{x} = t \Rightarrow x = t^2 \Rightarrow dx = 2t\,dt$

$\Rightarrow \int \frac{(t+1)(t^4-t)2t\,dt}{(t^3+t^2+t)} \Rightarrow \int \frac{2t^2(t+1)(t^3-1)dt}{t(t^2+t+1)}$

$\Rightarrow \int \frac{2t(t+1)(t-1)(t^2+t+1)}{(t^2+t+1)}dx \Rightarrow 2\int (t^3-t)dt$

$\Rightarrow \frac{2t^4}{4} - \frac{2t^2}{2} + c \Rightarrow \frac{x^2}{2} - x + c$

Sol.24 $\int\left[\sin^2\left(\frac{9\pi}{8}+\frac{x}{4}\right)-\sin^2\left(\frac{7\pi}{8}+\frac{x}{4}\right)\right]dx$

$\Rightarrow \int \sin\left(2\pi+\frac{x}{2}\right).\sin\frac{\pi}{4}dx \quad \left[\because \sin^2 A-\sin^2 B = \sin(A+B).\sin(A-B)\right]$

$\Rightarrow -\frac{1}{\sqrt{2}}.\,2.\cos\frac{x}{2} + C \Rightarrow -\sqrt{2}\cos\frac{x}{2} + c$

Sol.25 $\int \frac{\cos 4x+1}{\cot x-\tan x}dx$

$\Rightarrow \int \frac{2\cos^2 x.\sin x\cos x}{(\cos^2 x-\sin^2 x)}dx$

$\Rightarrow \int \frac{2\cos^2 2x.\sin x.\cos x}{\cos 2x}dx$

$\Rightarrow \int \cos 2x \sin 2x\,dx \Rightarrow \frac{1}{2}\int \sin 4x dx$

$\Rightarrow \frac{-1}{8}\cos 4x + c$

Sol.26

Given: $g'(x^2) = x^3$, $\forall\, x > 0$ & $g(1) = 1$

$g(4) = ?$

put $x^2 = t$ $\quad\therefore g'(t) = t^{3/2}$

$\Rightarrow \int g'(t)\,dt = \int t^{3/2}dt \Rightarrow g(t) = \frac{2}{5}t^{5/2} + c$

Now, $g(1) = 1 \Rightarrow c = \frac{3}{5}$

$\therefore g(4) = \frac{2}{5}.(32) + \frac{3}{5} = \frac{64+3}{5} \Rightarrow g(4) = \frac{67}{5}$

Sol.27 $\int\left[\sin\alpha\sin(x-\alpha)+\sin^2\left(\frac{x}{2}-\alpha\right)\right]dx$

$\Rightarrow \sin\alpha \int \sin(x-\alpha)dx + \int \frac{1-\cos(x-2\alpha)}{2}dx$

$\Rightarrow -\sin\alpha.\cos(x-\alpha) + \frac{1}{2}x - \frac{1}{2}\sin(x-2\alpha) + c$

$\Rightarrow \frac{1}{2}(x-\sin x) + c$

Sol.28 $\int \frac{\sin 2x+\sin 5x-\sin 3x}{\cos x+1-2\sin^2 2x}dx$

$\Rightarrow \int \frac{\sin 2x+2\sin x\cos 4x}{\cos x+\cos 4x}dx$

$\Rightarrow \int \frac{2\sin x(\cos x+\cos 4x)}{\cos x+\cos 4x}dx \Rightarrow -2\cos x + c$

Sol.29 $\int\left[\frac{\cot^2 2x-1}{2\cot 2x}-\cos 8x\cot 4x\right]dx$

$\Rightarrow \int\left[\frac{\cos^2 2x-\sin^2 2x}{2\cos^2 x\,\sin^2 x}-\frac{\cos 8x.\cos 4x}{\sin 4x}\right]dx$

$\Rightarrow \int \frac{\cos 4x}{\sin 4x}[1-\cos 8x]dx \Rightarrow \int \frac{\cos 4x}{\sin 4x}.\,2\sin^2 4x\,dx$

$\Rightarrow \int 2\cos 4x\sin 4x dx \Rightarrow \int \sin 8x dx$

$\Rightarrow \frac{-1}{8}\cos 8x + c$

Sol.30 $\Rightarrow \int \frac{\cos^4 x-\sin^4 x}{\sqrt{1+\cos 4x}}dx (\cos 2x > 0)$

$\Rightarrow \int \frac{(\cos^2 x-\sin^2 x}{\sqrt{1+\cos 4x}}dx$

$\Rightarrow \int \frac{\cos 2x}{\sqrt{2}\cos 2x}dx \quad [\because \cos 2x > 0] \Rightarrow \frac{x}{\sqrt{2}} + c$

Sol.31 $\int \frac{2x^3+3x^2+4x+5}{2x+1}dx$

$\Rightarrow$ Upon dividing,

$\Rightarrow \int\left(x^2+x+\frac{3}{2}\right)dx + \frac{7}{2}\int \frac{dx}{2x+1}$

$\Rightarrow \frac{x^3}{3} + \frac{x^2}{2} + \frac{3}{2}x + \frac{7}{4}\ln|2x+1| + c$

Sol.32 $\int \frac{(x^2+\sin^2 x)\sec^2 x}{1+x^2}dx$

$\Rightarrow \int \frac{(x^2+1\cos^2 x)\sec^2 x}{1+x^2}dx$

$\Rightarrow \int \sec^2 x\,dx - \int \frac{dx}{1+x^2} \Rightarrow \tan x - \tan^{-1}x + c$

Sol.33 $\int \frac{dx}{\sqrt{9-16x^2}}$

$\Rightarrow \frac{1}{4}\int \frac{dx}{\sqrt{\frac{9}{16}-x^2}} \Rightarrow \frac{1}{4}\sin^{-1}\left(\frac{4}{3}x\right) + c$

Sol.34 $\int \frac{dx}{25+4x^2}$

$\Rightarrow \frac{1}{4}\int \frac{dx}{\frac{25}{4}x^2} \Rightarrow \frac{1}{4}\cdot\frac{2}{5}\tan^{-1}\left(\frac{2x}{5}\right) + c$

$\Rightarrow \frac{1}{10}\tan^{-1}\left(\frac{2x}{5}\right) + c$

Sol.35 $\int \frac{2x+3}{3x+2}dx \Rightarrow$ upon dividing,

$\Rightarrow \frac{2}{3}\int dx + \frac{5}{3}\int \frac{dx}{3x+2} \Rightarrow \frac{2}{3}x + \frac{5}{9}\ln|3x+2| + c$

Sol.36 $\int \frac{dx}{1+\sin x}$

$\Rightarrow \int \frac{dx}{1+\sin x}\cdot\frac{(1-\sin x)}{(1-\sin x)} \Rightarrow \int \frac{1-\sin x}{\cos^2 x}dx$

$\Rightarrow \int \sec^2 x\,dx - \int \tan x\sec x\,dx$

$\Rightarrow \tan x - \sec x + c$

Sol.37 $\int \frac{\cos 8x - \cos 7x}{1+2\cos 5x}dx$

$\Rightarrow \int \frac{2\sin\frac{15x}{2}\sin\frac{x}{2}}{1+2\left(1-2\sin^2\frac{5x}{2}\right)}dx$

$\Rightarrow \int \frac{2\sin\frac{15x}{2}\sin\frac{x}{2}\sin\frac{5x}{2}}{\left(3\sin\frac{5x}{2}-4\sin^3\frac{5x}{2}\right)}dx$

$\Rightarrow \int 2\sin\frac{x}{2}\sin\frac{5x}{2}dx \Rightarrow \frac{\sin 3x}{3} - \frac{\sin 2x}{2} + c$

Sol.38 $\int \frac{2+3x^2}{x^2(1+x^2)}dx$

$\Rightarrow \int\left[\frac{2+2x^2}{x^2(1+x^2)} + \frac{x^2}{x^2(1+x^2)}\right]dx$

$\Rightarrow \int\left[\frac{2}{x^2} + \frac{1}{1+x^2}\right]dx \Rightarrow \frac{-2}{x} + \tan^{-1} + c$

Sol.39 $\int \frac{(\sin 2x)-(\sin 2k)}{\sin x - \sin k + \cos x - \cos k}$

$\int \frac{(\sin x+\cos x)^2 - 1 - (\sin k+\cos k)^2 + 1}{(\sin x+\cos x)-(\sin k+\cos k)}dx$

$\Rightarrow \int (\sin x + \cos x + \sin k + \cos k)dx$

$\Rightarrow (-\cos x + \sin x) + (\sin k + \cos k)\,x + c$

Sol.40 $\int \frac{x^2+3}{x^6(x^2+1)}dx$

$\Rightarrow \int\left[\frac{x^2+1}{x^6(x^2+1)} + \frac{2}{x^6(x^2+1)}\right]dx$

$\Rightarrow \int\left[\frac{1}{X^6} + \frac{2(1+X^6)}{X^6(X^2+1)} - \frac{2X^6}{X^6(X^2+1)}\right]dx$

$\Rightarrow \int\left[\frac{1}{x^6} + \frac{2(1+x^2)(x^4-x^2+1)}{x^6(x^2+1)} - \frac{2}{x^2+1}\right]dx$

$\Rightarrow \int\left[3x^{-6} + 2x^{-2} - 2x^{-4} - \frac{2}{x^2+1}\right]dx$

$\Rightarrow \frac{-3}{5x^5} + \frac{2}{3x^3} - \frac{2}{x} - 2\tan^{-1}x + c$

Sol.41 $\int \sin x\cos x\cos 2x\cos 4x\,dx$

$\Rightarrow \frac{1}{2}\int \sin 2x.\cos 2x.\cos 4x dx$

$\Rightarrow \frac{1}{4}\int \sin 4x.\cos 4x dx$

$\Rightarrow \frac{1}{8}\int \sin 8x\,dx \Rightarrow \frac{-1}{64}\cos 8x + c$

Sol.42 $\int x^x ln(ex)dx$

$\Rightarrow$ put $x^x = t \Rightarrow x\log_e x = \log_e t \Rightarrow (1+\log_e x)\,dx = \frac{1}{t}dt$

$\Rightarrow \int t.\frac{1}{t}dt \Rightarrow \int dt \Rightarrow t + C \Rightarrow x^x + c$